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Exact vacuum energy of orbifold lattice theories

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Abstract: We investigate the orbifold lattice theories constructed from supersymmetric Yang-Mills matrix theories (mother theories) with four and eight supercharges. We show that the vacuum energy of these theories does not receive any quantum correction perturbatively.

Keywords: Lattice Quantum Field Theory, Lattice Gauge Field Theories, BRST Symmetry, M(atrix) Theories.

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Contents

1. Introduction

Recently, there has been a rapid development in supersymmetric lattice gauge theories. A systematic way to construct supersymmetric lattice formulations is developed in [1−4], where a space-time lattice is generated by an orbifold projection of a supersymmetric Yang-Mills matrix theory (mother theory), and a lattice spacing is introduced by "deconstruction" [5]. By choosing the orbifold projection properly, one can make at least one supercharge or BRST charge preserved on the lattice. These formulations are further analysed in [6−10].[1] A prescription to generate a lattice theory from a topologically twisted continuum supersymmetric gauge theory is proposed by Catterall [12−14]. In these formulations, the BRST charge of the continuum theory is preserved on the lattice. A characteristic feature of these formulations is that all the degrees of freedom on the lattice except for site variables are doubled by a complexification and the path-integral is performed along "the real line". Numerical simulations are carried out for the model of two-dimensional $\mathcal{N} = (2, 2)$ supersymmetric gauge theory [15], which reproduce the Ward-Takahashi identities in fairly good accuracy. Other formulations constructed from topologically twisted supersymmetric gauge theories are developed by Sugino [16−19], where it is shown that the BRST transformation for the continuum fields can also be defined for lattice variables. The lattice action is straightforwardly generated from the Q-exact form of the continuum action by replacing all the fields by the lattice variables. A common feature of the above three formulations is that they possess at least one preserved supercharge or BRST charge. Alternative approach (the link approach) has been developed in [20−22], where it is claimed that all the supersymmetry of the continuum theory is preserved on the lattice. They first explicitly construct a supersymmetry algebra on a lattice and next make a lattice action based on the algebra, although there are some discussions on this approach [23, 24]. For conventional but useful approaches to supersymmetric lattice gauge theories, see [25−31]

[1] For a nice review, see [11].

in which the theories do not have any supersymmetry on a lattice but they flow to supersymmetric theories without fine-tuning because of a discrete chiral symmetry on the lattice. See also [32] for a recent lattice approach to two-dimensional $\mathcal{N} = (2,2)$ supersymmetric gauge theory.

The above seemingly different supersymmetric lattice formulations with a supercharge on the lattice are related to the orbifold lattice theories. In fact, the prescription given by Catterall can be reproduced using the orbifolding procedure [33]. Sugino's formulations can be obtained from Catterall's formulations by restricting the degrees of freedom of the complexified fields with preserving the supercharge [34]. Furthermore, the formulations given by the link approach have been shown to be equivalent to those given by orbifolding [35]. In this sense, it seems important to examine quantum mechanical properties of the orbifold lattice theories. In the next section, we examine the vacuum energy of the orbifold lattice theories constructed from $Q = 4$ and $Q = 8$ mother theories. We show that the vacuum energy exactly vanishes to all orders of the perturbation theory and the flat directions of these theories are never lifted up by any perturbative effect. The final section is devoted to conclusion and discussion.

2. Quantum corrections to vacuum energy

2.1 Orbifold lattice theories from $Q = 4$ mother theory

As discussed in detail in [1–4], an orbifold lattice theory is obtained by performing an appropriate orbifold projection to a supersymmetric Yang-Mills matrix theory (mother theory) followed by deconstruction, that is, by expanding the orbifolded matrix theory around a classical vacuum. Let us start with the orbifold lattice theories constructed from the dimensionally reduced four-dimensional $\mathcal{N} = 1$ supersymmetric Yang-Mills theory [2]. As discussed in [10], the lattice gauge theory obtained from this mother theory is essentially unique to be a lattice formulation for two-dimensional $\mathcal{N} = (2,2)$ supersymmetric Yang-Mills theory.[2] The action of the orbifolded matrix theory (before deconstruction) is given by

$$\begin{aligned}
S_{\text{orb}} = \frac{1}{g^2}\text{Tr}\sum_{\mathbf{n}\in\mathbb{Z}_N^2}\Bigg(&\frac{1}{4}\Big|z_m(\mathbf{n})z_n(\mathbf{n}+\mathbf{e}_m) - z_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n)\Big|^2 \\
&+\frac{1}{8}\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m)z_m(\mathbf{n}-\mathbf{e}_m)\Big)^2 \\
&+\psi_m(\mathbf{n})\Big(\bar{z}_m(\mathbf{n})\eta(\mathbf{n}) - \eta(\mathbf{n}+\mathbf{e}_m)\bar{z}_m(\mathbf{n})\Big) \\
&-\frac{1}{2}\chi_{mn}(\mathbf{n})\Big(z_m(\mathbf{n})\psi_n(\mathbf{n}+\mathbf{e}_n) - \psi_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n) - (m\leftrightarrow n)\Big)\Bigg),
\end{aligned} \tag{2.1}$$

where $m, n = 1, 2$, $\mathbf{e}_m$ are two linearly independent integer valued two-vectors, and all the fields are complex matrices with the size M. Although this action does not contain any lattice spacing nor kinetic terms, we can regard it as a lattice action by identifying $\mathbf{n}$ as the label of a site on a two-dimensional square lattice with the size N. In this sense, the

[2] In this paper, we restrict ourselves to consider gauge theories in d dimensional space-time with $d \geq 2$.

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variables $z_m(\mathbf{n})$ and $\bar{z}_m(\mathbf{n})$ are bosonic fields living on the links $(\mathbf{n}, \mathbf{n}+\mathbf{e}_m)$ and $(\mathbf{n}+\mathbf{e}_m, \mathbf{n})$, respectively, and $\eta(\mathbf{n})$, $\psi_m(\mathbf{n})$ and $\chi_{12}(\mathbf{n}) = -\chi_{21}(\mathbf{n})$ are fermionic fields living on the site $\mathbf{n}$, the link $(\mathbf{n}, \mathbf{n}+\mathbf{e}_m)$ and the link $(\mathbf{n}+\mathbf{e}_1+\mathbf{e}_2, \mathbf{n})$, respectively. Note the action (2.1) is invariant under a $\mathrm{U}(M)$ "gauge transformation" $z_m(\mathbf{n}) \to g^{-1}(\mathbf{n}) z_m(\mathbf{n}) g(\mathbf{n}+\mathbf{e}_m)$ $(g(\mathbf{n}) \in \mathrm{U}(M))$, and so on. As mentioned above, kinetic terms and a lattice spacing a are introduced by expanding $z_m(\mathbf{n})$ and $\bar{z}_m(\mathbf{n})$ as

$$z_m(\mathbf{n}) = \frac{1}{a}\mathbf{1}_M + z'_m(\mathbf{n}), \quad \bar{z}_m(\mathbf{n}) = \frac{1}{a}\mathbf{1}_M + \bar{z}'_m(\mathbf{n}), \tag{2.2}$$

then we obtain a lattice formulation for two-dimensional $\mathcal{N} = (2,2)$ supersymmetric Yang-Mills theory. Since the potential terms of this theory are given by

$$\frac{1}{4}\Big|z'_m(\mathbf{n})z'_n(\mathbf{n}+\mathbf{e}_m) - z'_n(\mathbf{n})z'_m(\mathbf{n}+\mathbf{e}_n)\Big|^2 + \frac{1}{8}\Big(z'_m(\mathbf{n})\bar{z}'_m(\mathbf{n}) - \bar{z}'_m(\mathbf{n}-\mathbf{e}_m)z'_m(\mathbf{n}-\mathbf{e}_m)\Big)^2, \tag{2.3}$$

the classical moduli space (the flat directions) of this theory is parametrized by the vacuum expectation values of $z'_m(\mathbf{n})$ and $\bar{z}'_m(\mathbf{n})$,

$$z'_m(\mathbf{n}) = \begin{pmatrix} b_m^1 & & \\ & \ddots & \\ & & b_m^M \end{pmatrix} \equiv b_m, \qquad \bar{z}'_m(\mathbf{n}) = \begin{pmatrix} \bar{b}_m^1 & & \\ & \ddots & \\ & & \bar{b}_m^M \end{pmatrix} \equiv \bar{b}_m, \tag{2.4}$$

with $b_m^i \in \mathbb{C}$ $(i = 1, \ldots, M)$ up to gauge transformations.

In this paper, we are interested in quantum corrections to this classical moduli space. To examine them, we will estimate the vacuum energy at the point (2.4) in the classical moduli space. Perturbatively, this is achieved by expanding the lattice action (after deconstruction) around the vacuum (2.4) and summing up all 1PI vacuum graphs. However, recalling that the action of the lattice gauge theory is obtained by substituting (2.2) into the action of the orbifolded matrix theory (2.1), we see that the same result is obtained by directly replacing $z_m(\mathbf{n})$ and $\bar{z}_m(\mathbf{n})$ in the action (2.1) with

$$\begin{aligned} z_m(\mathbf{n}) &\to z_m(\mathbf{n}) + \frac{1}{a}\mathbf{1}_M + b_m \equiv z_m(\mathbf{n}) + a_m, \\ \bar{z}_m(\mathbf{n}) &\to \bar{z}_m(\mathbf{n}) + \frac{1}{a}\mathbf{1}_M + \bar{b}_m \equiv \bar{z}_m(\mathbf{n}) + \bar{a}_m, \end{aligned} \tag{2.5}$$

respectively. In the following calculation, we will use this notation and estimate the vacuum energy as a function of a_m^i $(i = 1, \ldots, M)$.

We first calculate the 1-loop vacuum energy. It is convenient to fix the gauge by imposing a gauge condition,

$$D_m^- \bar{z}_m(\mathbf{n}) - \bar{D}_m^- z_m(\mathbf{n}) = 0, \tag{2.6}$$

where

$$D_m^- f(\mathbf{n}) \equiv a_m f(\mathbf{n}) - f(\mathbf{n}-\mathbf{e}_m) a_m, \qquad \bar{D}_m^- f(\mathbf{n}) \equiv -\bar{a}_m f(\mathbf{n}-\mathbf{e}_m) + f(\mathbf{n})\bar{a}_m. \tag{2.7}$$

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For the purpose of the later discussion, we also define

$$D_m^+ f(\mathbf{n}) \equiv a_m f(\mathbf{n}+\mathbf{e}_m) - f(\mathbf{n}) a_m, \qquad \bar{D}_m^+ f(\mathbf{n}) \equiv -\bar{a}_m f(\mathbf{n}) + f(\mathbf{n}+\mathbf{e}_m)\bar{a}_m. \tag{2.8}$$

By introducing gauge fixing terms and FP ghost fields corresponding to the gauge condition (2.6) in a standard way, the second-order action is obtained as

$$\begin{aligned} S^{(2)} = \frac{1}{g^2} \mathrm{Tr} \sum_{\mathbf{n}\in\mathbb{Z}_N^2} \Bigg(& \frac{1}{2}\bar{D}_n^- z_m(\mathbf{n}) D_n^- z_m(\mathbf{n}) + \frac{1}{2}\bar{D}_n^- B(\mathbf{n}) D_n^- C(\mathbf{n}) \\ & + \eta(\mathbf{n})\bar{D}_m^- \psi_m(\mathbf{n}) - \frac{1}{2}\chi_{mn}(\mathbf{n})\Big(D_m^+\psi_n(\mathbf{n}) - D_n^+\psi_m(\mathbf{n})\Big)\Bigg), \end{aligned} \tag{2.9}$$

where $B(\mathbf{n})$ and $C(\mathbf{n})$ are FP ghost fields. By integrating over the fields, we get the 1-loop contribution to the partition function as[3]

$$\begin{aligned} Z\Big|_{1-\mathrm{loop}} &= \int \prod_{\mathbf{n}} d\Phi(\mathbf{n}) e^{-S^{(2)}[\Phi(\mathbf{n})]} \\ &= \frac{\det\Delta}{\det\Delta} = 1, \end{aligned} \tag{2.10}$$

where $\Delta \equiv \sum_m \bar{D}_m^+ D_m^-$ is the Laplacian and the lattice variables have been abbreviated as $\Phi(\mathbf{n})$. The denominator of the second line comes from the contributions from the bosonic fields and the ghost fields and the numerator comes from the fermionic fields. The result (2.10) means that the vacuum energy is equal to zero and the classical flat directions remain flat at the 1-loop level. Note that the same calculation is carried out at the origin of the moduli space in [8]. We can reproduce it by setting $b_m^i = 0$ (or $a_m^i = 1/a$) in our calculation.

One might think that, even though the 1-loop contribution to the vacuum energy is zero, higher-loop contributions would give non-trivial corrections to the vacuum energy, since the supersymmetry is almost broken except for the only one preserved supercharge (or BRST charge). However, we can show that it is not the case and the above 1-loop result is exact. The key point is that the action (2.1) can be written in a Q-exact form [2]:

$$\begin{aligned} S_{\mathrm{orb}} = \frac{1}{2g^2} \mathrm{Tr} \sum_{\mathbf{n}\in\mathbb{Z}_N^2} Q\Bigg(& \eta(\mathbf{n})\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m) z_m(\mathbf{n}-\mathbf{e}_m) + d(\mathbf{n})\Big) \\ & - \chi_{mn}(\mathbf{n})\Big(z_m(\mathbf{n}) z_n(\mathbf{n}+\mathbf{e}_n) - z_n(\mathbf{n}) z_m(\mathbf{n}+\mathbf{e}_n)\Big)\Bigg), \end{aligned} \tag{2.11}$$

[3] In this calculation, the constant modes are treated by shifting the difference operators (2.7) and (2.8) as $D_m^\pm \to D_m^\pm + i\mu$, which corresponds to adding mass terms as done in [2]. Although this modification breaks the BRST symmetry, the final result of the following discussion still holds in the limit of $\mu \to 0$ since the breaking of the symmetry is soft.

where Q is a BRST charge that acts on the fields as

$$\begin{aligned}
Qz_m(\mathbf{n}) &= \psi_m(\mathbf{n}), \quad Q\bar{z}_m(\mathbf{n}) = 0, \\
Qd(\mathbf{n}) &= \psi_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m)\psi_m(\mathbf{n}-\mathbf{e}_m), \\
Q\eta(\mathbf{n}) &= \frac{1}{4}\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m)z_m(\mathbf{n}-\mathbf{e}_m) - d(\mathbf{n})\Big), \\
Q\chi_{mn}(\mathbf{n}) &= \frac{1}{2}\Big(\bar{z}_m(\mathbf{n}+\mathbf{e}_n)\bar{z}_n(\mathbf{n}) - \bar{z}_n(\mathbf{n}+\mathbf{e}_m)\bar{z}_m(\mathbf{n})\Big),
\end{aligned} \tag{2.12}$$

and $d(\mathbf{n})$ is an auxiliary bosonic field which makes Q be nilpotent off-shell. Recalling the discussion in topological field theory [36], we see that the partition function of this theory does not depend on the coupling constant g. In fact, if we write the partition function as $Z(g) = \int \mathcal{D}\Phi e^{\frac{1}{g^2}Q\Xi[\Phi]}$, the derivative of the partition function by g gives

$$\frac{d}{dg}Z(g) \propto \Big\langle Q\Xi[\Phi]\Big\rangle = 0, \tag{2.13}$$

where $\langle\mathcal{O}\rangle$ denotes the expectation value of an operator $\mathcal{O}$ and we have used the fact that, as long as the BRST symmetry is not broken spontaneously, the expectation value of a Q-exact operator vanishes when the action is Q-exact.[4] This means that the partition function evaluated in the weak coupling limit, that is, the 1-loop result given above is exact. In particular, we can expect that all the higher-loop contributions to the vacuum energy vanish.

Note that one might think that the partition function given above expresses not the vacuum energy but the Witten index of the theory since we impose the periodic boundary condition to the fermionic fields in the time direction. Although it is actually the case, the boundary conditions do not affect the perturbative contributions in the limit that the period of the time direction goes to infinity. Therefore we can conclude that there is no perturbative correction to the vacuum energy from (2.13).[5]

Another note is that we can apply the same analysis to a deformed theory given by[6]

$$\begin{aligned}
S_{\mathrm{orb}} = \frac{1}{2g^2}\mathrm{Tr}\sum_{\mathbf{n}\in\mathbb{Z}_N^2} Q\Bigg(&\eta(\mathbf{n})\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m)z_m(\mathbf{n}-\mathbf{e}_m) + d(\mathbf{n})\Big) \\
&- \beta\chi_{mn}(\mathbf{n})\Big(z_m(\mathbf{n})z_n(\mathbf{n}+\mathbf{e}_n) - z_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n)\Big)\Bigg),
\end{aligned} \tag{2.14}$$

where $\beta \in \mathbb{R}$ and the BRST transformation is given by (2.12). By construction, this deformation does not spoil the Q-exactness of the action and it becomes identical with the original orbifolded matrix theory (2.1) by setting $\beta = 1$. We can show that the vacuum

[4] For a discussion on spontaneous breaking of supersymmetry for two-dimensional $\mathcal{N} = (2,2)$ theories, see [37].

[5] The author would like to thank to H. Suzuki for discussing this point.

[6] The physical interpretation of this deformation is still unclear. In fact, the continuum limit of this deformed theory is not Lorentz invariant, though it has a BRST symmetry generated by Q. The author would like to thank M. Ünsal for pointing it out.

energy of this deformed theory also vanishes at the 1-loop level. Therefore, repeating the same argument above, we can conclude that there is no perturbative correction to the vacuum energy of this theory.

2.2 Orbifold lattice theories from $Q=8$ mother theory

Next we consider the lattice theories constructed from the mother theory with eight supercharges, that is, the dimensionally reduced six-dimensional $\mathcal{N}=1$ supersymmetric Yang-Mills theory [3]. By performing an orbifold projection to the mother theory, we obtain the action of the orbifolded matrix theory [10]:

$$\begin{aligned}S_{\rm orb} = \frac{1}{g^2}\mathrm{Tr}\sum_{\mathbf{n}\in\mathbb{Z}_N^d}\Bigg(&\frac{1}{4}\Big|z_m(\mathbf{n})z_n(\mathbf{n}+\mathbf{e}_m)-z_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n)\Big|^2 \\ &+\frac{1}{8}\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n})-\bar{z}_m(\mathbf{n}-\mathbf{e}_m)z_m(\mathbf{n}-\mathbf{e}_m)\Big)^2 \\ &-\psi_m(\mathbf{n})\Big(\bar{z}_m(\mathbf{n})\eta(\mathbf{n})-\eta(\mathbf{n}+\mathbf{e}_m)\bar{z}_m(\mathbf{n})\Big) \\ &+\frac{1}{2}\xi_{mn}(\mathbf{n})\Big(z_m(\mathbf{n})\psi_n(\mathbf{n}+\mathbf{e}_m)-\psi_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n)-(m\leftrightarrow n)\Big) \\ &-\frac{1}{2}\chi_{lmn}(\mathbf{n})\Big(\bar{z}_l(\mathbf{n}+\mathbf{e}_m+\mathbf{e}_n)\xi_{mn}(\mathbf{n})-\xi_{mn}(\mathbf{n}+\mathbf{e}_l)\bar{z}_l(\mathbf{n})\Big)\Bigg),\end{aligned} \tag{2.15}$$

where $l,m,n=1,2,3$, $\mathbf{e}_m$ are integer valued three-component vectors, d is the number of linearly independent vectors in $\{\mathbf{e}_m\}$, and again we assume that all the fields are complex matrices with the size M. Note that d is the maximal dimensionality of the lattice theory obtained after deconstruction. The fields $z_m(\mathbf{n})$ and $\bar{z}_m(\mathbf{n})$ are bosonic fields living on links $(\mathbf{n},\mathbf{n}+\mathbf{e}_m)$ and $(\mathbf{n}+\mathbf{e}_m,\mathbf{n})$, respectively, and $\eta(\mathbf{n})$, $\psi_m(\mathbf{n})$, $\xi_{mn}(\mathbf{n})$ and $\chi_{lmn}(\mathbf{n})$ are fermionic fields on the site $\mathbf{n}$, the link $(\mathbf{n},\mathbf{n}+\mathbf{e}_m)$, the link $(\mathbf{n}+\mathbf{e}_m+\mathbf{e}_n,\mathbf{n})$ and the link $(\mathbf{n},\mathbf{n}+\mathbf{e}_l+\mathbf{e}_m+\mathbf{e}_n)$, respectively. The fields $\xi_{mn}(\mathbf{n})$ and $\chi_{lmn}(\mathbf{n})$ are antisymmetric in terms of a permutation of the indices.

In this case, we can construct several kinds of supersymmetric lattice gauge theories with a different dimensionality, with a different number of preserved supercharges and with a different lattice structure by changing the vectors $\mathbf{e}_m$ and the number of bosonic fields to shift as (2.2) [10]. Recalling the discussion around (2.5), however, we can estimate the vacuum energy of these theories at once by directly expanding the orbifolded matrix theory (2.15) around

$$z_m(\mathbf{n})=\begin{pmatrix}a_m^1 & & \\ & \ddots & \\ & & a_m^M\end{pmatrix}\equiv a_m,\qquad \bar{z}_m(\mathbf{n})=\begin{pmatrix}\bar{a}_m^1 & & \\ & \ddots & \\ & & \bar{a}_m^M\end{pmatrix}\equiv \bar{a}_m. \tag{2.16}$$

By fixing the gauge by the gauge condition (2.6), we obtain the second-order action,

$$S^{(2)}=\frac{1}{g^2}\mathrm{Tr}\sum_{\mathbf{n}\in\mathbb{Z}_N^d}\Bigg(\frac{1}{2}\bar{D}_n^- z_m(\mathbf{n})D_n^- z_m(\mathbf{n})+\frac{1}{2}\bar{D}_n^- B(\mathbf{n})D_n^- C(\mathbf{n})+\eta(\mathbf{n})\bar{D}_m^-\psi_m(\mathbf{n})$$

$$-\frac{1}{2}\xi_{mn}(\mathbf{n})\Big(D^+_m\psi_n(\mathbf{n}) - D^+_n\psi_m(\mathbf{n})\Big) + \frac{1}{2}\xi_{mn}(\mathbf{n})\bar{D}^-_l\chi_{lmn}(\mathbf{n})\Bigg). \quad (2.17)$$

From this expression, it is easy to show that the 1-loop contribution to the vacuum energy vanishes again.

As for the case of the $Q = 4$ orbifold lattice theories, the lattice theory (2.15) possesses a BRST charge Q that acts on the fields as [3]

$$\begin{aligned} Qz_m(\mathbf{n}) &= \psi_m(\mathbf{n}), \quad Q\bar{z}_m(\mathbf{n}) = 0, \\ Qd(\mathbf{n}) &= \psi_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m)\psi_m(\mathbf{n}-\mathbf{e}_m), \\ Q\eta(\mathbf{n}) &= \frac{1}{4}\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m)z_m(\mathbf{n}-\mathbf{e}_m) - d(\mathbf{n})\Big), \\ Q\xi_{mn}(\mathbf{n}) &= \frac{1}{2}\Big(\bar{z}_m(\mathbf{n}+\mathbf{e}_n)\bar{z}_n(\mathbf{n}) - \bar{z}_n(\mathbf{n}+\mathbf{e}_m)\bar{z}_m(\mathbf{n})\Big), \\ Q\chi_{lmn}(\mathbf{n}) &= 0, \end{aligned} \quad (2.18)$$

where $d(\mathbf{n})$ is again an auxiliary field to make Q nilpotent off-shell. Here we can extend (2.18) by supplementing the fields with an additional bosonic field $f_{lmn}(\mathbf{n})$ satisfying

$$Qf_{lmn}(\mathbf{n}) = \chi_{lmn}(\mathbf{n}). \quad (2.19)$$

Then the action of the orbifolded matrix theory (2.15) can be equivalently expressed in a Q-exact form:

$$\begin{aligned} S_{\text{orb}} = \frac{1}{2g^2}\text{Tr}\sum_{\mathbf{n}\in\mathbb{Z}^d_N} Q\Bigg(&\eta(\mathbf{n})\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m)z_m(\mathbf{n}-\mathbf{e}_m) + d(\mathbf{n})\Big) \\ &- \chi_{mn}(\mathbf{n})\Big(z_m(\mathbf{n})z_n(\mathbf{n}+\mathbf{e}_n) - z_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n)\Big) \\ &- \frac{1}{2}f_{lmn}(\mathbf{n})\Big(\bar{z}_l(\mathbf{n}+\mathbf{e}_m+\mathbf{e}_n)\xi_{mn}(\mathbf{n}) - \xi_{mn}(\mathbf{n}+\mathbf{e}_l)\bar{z}_l(\mathbf{n})\Big)\Bigg). \end{aligned} \quad (2.20)$$

Note that, although the partition function diverges by integration over $f_{lmn}(\mathbf{n})$, it is irrelevant for the vacuum energy. Therefore, the 1-loop result given above is shown to be exact by repeating the argument in the previous subsection, and the vacuum energy is expected to be zero in all order of the perturbative expansion.

In summary, we can conclude that *the flat directions of the orbifold lattice theories constructed from the mother theory with four and eight supersymmetries do not receive any quantum correction perturbatively.*

3. Conclusion and discussion

In this paper, we examined quantum corrections to the classical moduli space of orbifold supersymmetric lattice theories constructed from the $Q = 4$ and $Q = 8$ mother theories.

We showed that the classical moduli space does not receive any quantum correction perturbatively, namely, the flat directions of these theories remain flat even if we take into account quantum effects. We also modified the action of the $Q = 4$ orbifolded matrix theory without spoiling the Q-exactness and showed that the classical moduli space of the deformed theory does not receive any perturbative correction either. Note that these results might be invalidated if the Q-symmetry were to be spontaneously broken. However, we can expect that it does not affect the perturbative results in any way since supersymmetry breaking is usually caused by a non-perturbative effect, if it is not broken at the tree level.

We conclude this paper by making some comments on other orbifold lattice theories. Let us first consider an orbifolded matrix theory,

$$\begin{aligned} S_{\rm orb} =& \frac{1}{g^2}\mathrm{Tr}\sum_{\mathbf{n}}\Bigg(\frac{1}{4}\Big|z_m(\mathbf{n})z_n(\mathbf{n}+\mathbf{e}_m) - z_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n)\Big|^2 \\ &+ \frac{1}{8}\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n}) - \bar{z}_m(\mathbf{n}-\mathbf{e}_m)z_m(\mathbf{n}-\mathbf{e}_m)\Big)^2 \\ &+ \eta(\mathbf{n})\Big(\bar{z}_m(\mathbf{n}+\mathbf{a}-\mathbf{e}_m)\psi_m(\mathbf{n}+\mathbf{a}-\mathbf{e}_m) - \psi_m(\mathbf{n}+\mathbf{a})\bar{z}_m(\mathbf{n}+\mathbf{a})\Big) \\ &- \frac{1}{2}\chi_{mn}(\mathbf{n})\Big(z_m(\mathbf{n})\psi_n(\mathbf{n}+\mathbf{e}_m) - \psi_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{a}_n) - (m\leftrightarrow n)\Big)\Bigg), \end{aligned} \tag{3.1}$$

where $\mathbf{e}_m$, $\mathbf{a}$, $\mathbf{a}_m$ and $\mathbf{a}_{12}$ are three-component vectors satisfying

$$\mathbf{a}+\mathbf{a}_m = \mathbf{e}_m, \quad \mathbf{a}_{12}+\mathbf{a}_m = -|\epsilon_{mn}|\mathbf{e}_n, \quad \mathbf{a}+\mathbf{a}_1+\mathbf{a}_2+\mathbf{a}_{12} = 0, \tag{3.2}$$

$z_m(\mathbf{n})$ and $\bar{z}_m(\mathbf{n})$ are the same bosonic fields as in (2.1) but $\eta(\mathbf{n})$, $\psi_m(\mathbf{n})$ and $\chi_{12}(\mathbf{n})$ are fermionic fields living on the links $(\mathbf{n},\mathbf{n}+\mathbf{a})$, $(\mathbf{n},\mathbf{n}+\mathbf{a}_m)$ and $(\mathbf{n}-\mathbf{a}_{12},\mathbf{n})$, respectively. In particular, we assume that any of the vectors $\mathbf{a}$, $\mathbf{a}_m$ and $\mathbf{a}_{12}$ is not zero. The action (3.1) has been first given in [21] and is shown to be obtained from $Q = 4$ mother theory by an orbifold projection with no preserved supercharge in any usual sense [35]. It is easy to show that the vacuum energy of this theory again vanishes at the 1-loop level. However, in this case, there seems to be no guarantee that higher-loop contributions to the vacuum energy vanish, since there is no usual BRST symmetry in this theory. It would be interesting, however, to investigate quantum corrections to this theory from the view point of the supersymmetry algebra on lattice discussed in [20, 21].

Interesting orbifold lattice theories are those constructed from $Q = 16$ mother theory [4], that is, IKKT matrix theory [38]. The action of the corresponding orbifolded matrix theory is written as

$$S_{\rm orb} = \frac{1}{g^2}\mathrm{Tr}\sum_{\mathbf{n}\in\mathbb{Z}_N^d}\Bigg(\frac{1}{4}\Big|z_m(\mathbf{n})z_n(\mathbf{n}+\mathbf{e}_m) - z_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n)\Big|^2 \tag{3.3}$$

$$
\begin{aligned}
&+\frac{1}{8}\Big(z_m(\mathbf{n})\bar{z}_m(\mathbf{n})-\bar{z}_m(\mathbf{n}-\mathbf{e}_m)z_m(\mathbf{n}-\mathbf{e}_m)\Big)^2\\
&-\psi_m(\mathbf{n})\Big(\bar{z}_m(\mathbf{n})\eta(\mathbf{n})-\eta(\mathbf{n}+\mathbf{e}_m)\bar{z}_m(\mathbf{n})\Big)\\
&+\frac{1}{2}\xi_{mn}(\mathbf{n})\Big(z_m(\mathbf{n})\psi_n(\mathbf{n}+\mathbf{e}_m)-\psi_n(\mathbf{n})z_m(\mathbf{n}+\mathbf{e}_n)-(m\leftrightarrow n)\Big)\\
&-\frac{1}{2}\epsilon_{mnpqr}\xi_{mn}(\mathbf{n})\Big(\bar{z}_p(\mathbf{n}+\mathbf{e}_q+\mathbf{e}_r)\xi_{pq}(\mathbf{n})-\xi_{pq}(\mathbf{n}+\mathbf{e}_p)\bar{z}_p(\mathbf{n})\Big)\Bigg),
\end{aligned}
$$

where $m,n=1,\dots,5$, $\mathbf{e}_m$ are five-component vectors satisfying $\sum_{m=1}^5 \mathbf{e}_m=0$ and d is the number of the linearly independent vectors in $\{\mathbf{e}_m\}$. Again the classical vacua are parametrized as (2.4) with $m=1,\dots 5$, and it is straightforward to show that the vacuum energy is zero at the 1-loop level. However, we cannot apply the same argument in the previous section since the last term of the action (3.3) is not Q-exact but Q-closed [4]. Thus, there is a possibility that the classical flat directions would be lifted up by quantum effects. In fact, from the viewpoint of the superstring theory, we can expect that non-trivial quantum corrections to the vacuum energy exist in this case. Recalling that the mother theory with sixteen supercharges is identical with the low energy effective theory on D-instantons on a ten-dimensional flat space-time, the orbifolded matrix theory (3.3) can be regarded as the low energy effective theory on D-instantons in the background of an orbifold.[7] In this interpretation, the background (2.4) can be regarded as the positions of D-instantons. The point is that this orbifold background breaks the supersymmetry on the ten-dimensional space-time, so (2.4) or (2.16) gives a non-BPS configuration of D-branes. Therefore, it seems that there should be some force between the separated D-instantons. In terms of the theory on the D-instantons, this means the classical flat directions parametrized by a_m are no longer flat if we take into account quantum corrections to the orbifolded matrix model. It would be interesting to analyse these theories along this way [40].

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